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Chiral glass ordering in the XY spin glass in four dimensions

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Abstract. The chiral-glass behaviour of the nearest-neighbour random-bond XY spin glass in four dimensions is studied by Monte Carlo simulations. A chiral-glass transition at $T_{cg} = 0.90 \pm 0.05$ is found by a finite-size scaling analysis of the results. The associated chiral correlation-length exponent is estimated to be $\nu_{cg} = 0.6 \pm 0.1$ and $\eta_{cg} \sim 0.25$. The values for the chiral critical temperature and the exponents are very similar to those recently reported for the spin-glass transition in this model. The results strongly suggest a simultaneous ordering of spin and chirality in four dimensions.

1. Introduction

Although it has been known since 1977 [1] that frustrated vector spin systems possess both reflectional and rotational symmetries, it is only fairly recently that the chiral-glass behaviour of vector spin glasses has been studied [2–9]. Whereas the continuous rotational symmetry is associated with the spins, it is the discrete twofold Ising-like reflectional symmetry which is associated with chirality. Much of the recent work has been motivated by the suggestion that the chiral-glass transition in vector spin glasses belongs to the same universality class as the Ising spin-glass transition. This would imply that the spin-glass transition observed in experiments may, in fact, be ‘chirality driven’ [7]. Consequently, one would have a chiral-glass phase with broken reflectional symmetry but preserved rotational symmetry. Recent numerical work by Kawamura [4, 7, 8] in three dimensions (3D) would appear to be consistent with this picture.

There is now convincing evidence that both XY [3–6, 10] and Heisenberg [6–8, 11] spin glasses exhibit conventional spin-glass ordering only at zero temperature for two dimensions (2D) and 3D. Domain-wall renormalization-group studies suggest chiral ordering for both XY [5] and Heisenberg [7] spin glasses in 2D also at zero temperature only. However, the values of the chiral- and spin-glass correlation-length exponents have been found to differ in both cases. Further evidence for a decoupling of the chiral and phase variables on long length scales has come from various Monte Carlo studies in 2D [2, 3], including very recent work using a vortex representation [9].

In 3D the results point to a finite-temperature chiral-glass transition both for XY [4–6] and Heisenberg [6–8] spin glasses (although the case is far from convincing for the latter). So it would appear that chiralities and spins have markedly different behaviour in both 2D and 3D.

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Very recent Monte Carlo simulations in four dimensions suggest a finite-temperature spin-glass transition for vector spin glasses [12,13]. This would imply that the lower critical dimensionality for vector spin glasses is less than 4. It is clearly of interest to see whether the difference in the behaviour of chiralities and spins extends into higher dimensions.

In this paper we present the results from Monte Carlo simulations of the chiral behaviour of the four dimensional random-bond XY spin glass. Using finite-size scaling and our earlier results for the spin-glass transition [12], we shall find evidence for a simultaneous ordering of spin and chirality in four dimensions (4D).

In section 2 we define the model and review the finite-size scaling technique used to analyse the data. Section 3 gives details about the simulations and the results are presented and discussed in section 4. The conclusion is given in section 5.

2. The model

The Hamiltonian for the model is given by

$$H = - \sum_{\langle i,j \rangle} J_{ij} \cos(\theta_i - \theta_j) \quad (1)$$

where $0 \leq \theta_i \leq 2\pi$ for all planar spins i and the summation runs over all nearest-neighbour pairs on a four-dimensional hypercubic lattice with $L \times L \times L \times L$ spins ($L = 2, 4$ and 6) with full periodic boundary conditions. The interactions, J_{ij} , are independent random variables selected from a binary ± 1 distribution. Throughout, the temperature is given in units of the nearest-neighbour interaction. We recently reported the results concerning the spin-glass transition for this model [12]. In this paper we concentrate on the chiral-glass transition.

The local chirality, κ_α , at a plaquette α consisting of four spins is defined by the scalar [2]

$$\kappa_\alpha = 2^{-3/2} \sum_{\alpha} \text{Sgn}(J_{ij}) \sin(\theta_i - \theta_j) \quad (2)$$

where the summation is performed over a directed (clockwise) closed contour along the four sides of the plaquette. Clearly, whereas $\kappa_\alpha = 0$ for an isolated unfrustrated plaquette, it is restricted to the values ± 1 for frustrated ones. Hence, chirality act as a ‘continuous’ Ising-like quantity. However, as its magnitude fluctuates with the temperature to a certain extent, we work with a root-mean-square value given by [2]

$$\bar{\kappa} = \sqrt{\frac{1}{N_d} \sum_{\alpha} [\langle \kappa_\alpha^2 \rangle_T]_J} \quad (3)$$

where $\langle \dots \rangle_T$ denotes a thermal average, $[\dots]_J$ indicates an average over the disorder and N_d , the total number of plaquettes for our four-dimensional lattice, is equal to $6L^4$.

At a chiral-glass transition one expects the chiral-glass susceptibility, χ_{cg} , to diverge, where

$$\begin{aligned} \chi_{cg} &= \frac{1}{N_d} \sum_{\alpha, \beta} [\langle \kappa_\alpha \kappa_\beta \rangle_T^2]_J \\ &= N_d q_{cg}^{(2)} \end{aligned} \quad (4)$$

and here the summation is with respect to all plaquettes α and β .

The chiral-glass order parameter, $q_{\text{cg}}^{(2)}$, defined above can be written in terms of the overlap between two replicas 1 and 2, namely

$$q_{\text{cg}}^{(2)} = [\langle q^2 \rangle_{\text{T}}]_J \tag{5}$$

where

$$q = \frac{1}{N_d} \sum_{\alpha} \kappa_{\alpha}^1 \kappa_{\alpha}^2. \tag{6}$$

In order to improve the analogy with Ising spins, we follow Kawamura [2] and work in the simulations with the reduced chiral-glass susceptibility

$$\tilde{\chi}_{\text{cg}} = \chi_{\text{cg}} / \bar{\kappa}^4. \tag{7}$$

Another key quantity studied in the simulations is the dimensionless Binder parameter defined by [14]

$$g_{\text{cg}}(L, T) = \frac{1}{2} \left[3 - \frac{q_{\text{cg}}^{(4)}}{(q_{\text{cg}}^{(2)})^2} \right] \tag{8}$$

and here

$$q_{\text{cg}}^{(4)} = [\langle q^4 \rangle_{\text{T}}]_J. \tag{9}$$

The Binder parameter is expected to scale as [14]

$$g_{\text{cg}}(L, T) = \bar{g}_{\text{cg}}(L^{1/\nu_{\text{cg}}}(T - T_{\text{cg}})) \tag{10}$$

where T_{cg} and ν_{cg} are the chiral-glass critical temperature and correlation-length exponent, respectively. The value of T_{cg} can be located by using the fact that the scaling function, \bar{g}_{cg} , and, hence, also g_{cg} are independent of L at the transition temperature. The chiral-glass correlation-length exponent then follows from a one-parameter scaling fit of the data for g_{cg} .

The finite-size scaling form for the reduced chiral-glass susceptibility is given by

$$\tilde{\chi}_{\text{cg}}(L, T) = L^{2-\eta_{\text{cg}}} \bar{\tilde{\chi}}_{\text{cg}}(L^{1/\nu_{\text{cg}}}(T - T_{\text{cg}})). \tag{11}$$

Here η_{cg} is the chiral critical-point decay exponent and $\bar{\tilde{\chi}}_{\text{cg}}$ is the scaling function.

The finite-size scaling form for the analogous spin-glass quantities can be found in [12].

3. Simulations

We now discuss the computer simulations (Jain [12] should be consulted for further technical details). In order to ensure that equilibrium has been achieved in the simulations, we use the technique of Bhatt and Young [14] whereby chiral-glass correlations ($q_{\text{cg}}^{(2)}$ and $q_{\text{cg}}^{(4)}$) computed from two replicas at the same time are required to agree with those from one replica at two different times.

At low temperatures the chiral degrees of freedom were more difficult to equilibrate than their spin counterparts, requiring approximately twice as many Monte Carlo steps. As a result, the lowest temperatures studied were $T = 0.8$ ($L = 6$), 0.7 ($L = 4$) and 0.3 ($L = 2$). We took disorder averages over many pairs of independent samples: $100 \sim 120$ ($L = 6$), $100 \sim 250$ ($L = 4$) and $250 \sim 700$ ($L = 2$) for each temperature. Most of the computational time (about 200 h of CPU time on a Cray J932) was taken up by the simulations for $L = 6$. (The processors of the Cray J932 are roughly only half the speed of those of the Cray YMP.) It is estimated that an additional 1000 h of CPU time would be required to obtain reliable data on a larger lattice such as $L = 8$. All computational studies performed to-date, including the present one, on four-dimensional spin glasses have been restricted to small lattices with $L \leq 6$ [12–14].

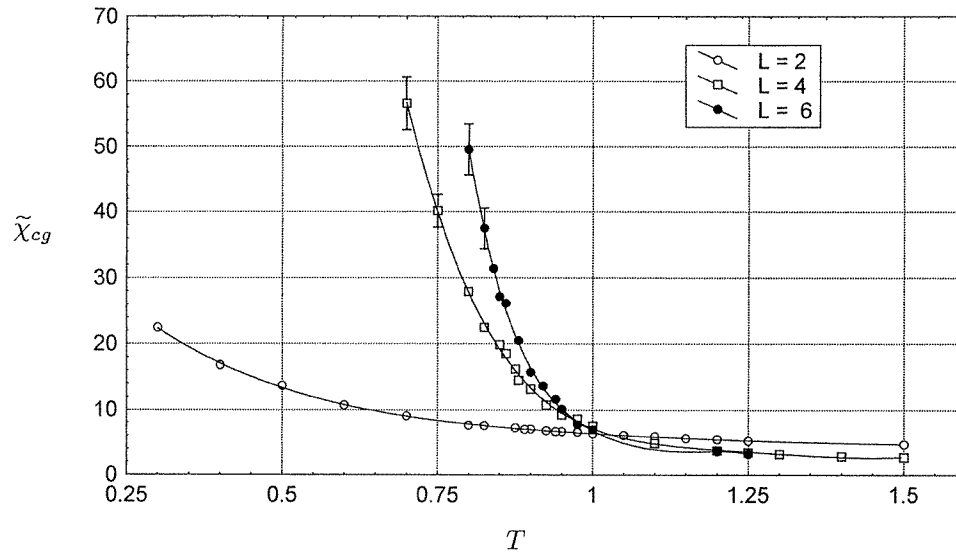


Figure 1. A plot of the reduced chiral-glass susceptibility, $\tilde{\chi}_{cg}$, against the temperature for $L = 2, 4$ and 6 (see equations (4) and (7) in the text). The curves are just to guide the eye.

4. Results

Figure 1 shows a plot of the reduced chiral-glass susceptibility against temperature for the three different lattices considered in this work. The statistical error bars have been estimated from the sample-to-sample fluctuations and, in most cases, are smaller than the size of the data points.

We see that the value of $\tilde{\chi}_{cg}$ remains quite low until $T \approx 1.0$ and then increases rapidly with the system size. In fact, whereas for low temperatures $\tilde{\chi}_{cg}$ is an increasing function of L , for higher temperatures it appears to actually decrease with increasing L . This suggests that the scaling regime is probably quite narrow. A very similar behaviour was recently found by Kawamura [4] in 3D.

In figure 2(a) we show a plot of g_{cg} against the temperature for $0.4 \leq T \leq 1.5$. The behaviour of the Binder parameter for high temperatures $T \geq 1.0$ indicates a disordered chiral phase. For $T \leq 1.0$ there is a sharp increase in g_{cg} , indicating a build up of chiral correlations. The increase is more noticeable for the larger lattices. We note that for $T \geq 1.0$ g_{cg} assumes negative values. A negative Binder parameter has also been seen for both XY [4] and Heisenberg [8] spin glasses in 3D. This feature would appear to be a consequence of the fact that $\kappa_\alpha = 0$ on unfrustrated plaquettes even in the *ordered* state.

Figure 2(b) displays on a much expanded scale the data in figure 2(a) in the vicinity of the important temperature region ($0.8 \leq T \leq 1.0$). We see that the curves appear to intersect at $T_{cg} \approx 0.9$. Furthermore, below T_{cg} the values corresponding to $L = 6$ are consistently above those for $L = 4$ and the curves clearly splay out. We estimate the chiral-glass transition temperature to be $T_{cg} = 0.90 \pm 0.05$. This value is very close to the spin-glass transition temperature ($T_{sg} = 0.95 \pm 0.15$) we recently reported for this model [12]. Our conclusion is based on the results for small lattices and it is highly desirable to obtain additional data for larger lattices to confirm our findings.

Further evidence for a finite-temperature transition comes from a scaling plot of the Binder parameter. By varying the value of ν_{cg} and considering values $0.85 \leq T_{cg} \leq 0.95$,

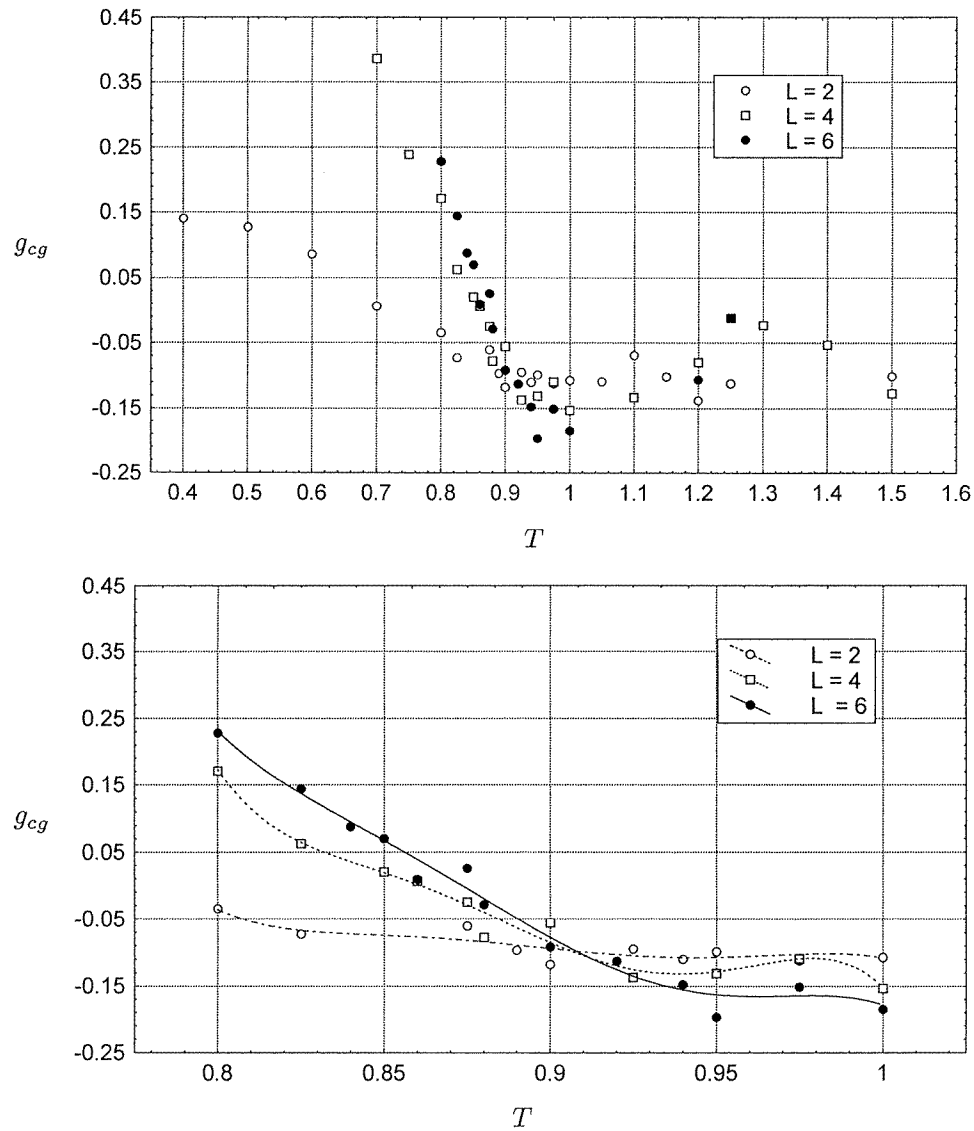


Figure 2. (a) A plot of the Binder parameter g_{cg} against the temperature for $L = 2, 4$ and 6 . (b) Same as in (a) but on a much expanded scale around the interesting temperature region ($T \approx 0.9$). The curves are just to guide the eye.

we estimate the chiral correlation-length exponent to be $\nu_{cg} = 0.6 \pm 0.1$. It should be noted that the error bar quoted here is simply an estimate that demarcates the range of values beyond which the data do not scale well. Figure 3 shows the data for g_{cg} against $L^{1/\nu_{cg}}(T - T_{cg})$ with $T_{cg} = 0.90$ and $\nu_{cg} = 0.6$. This scaling plot is far better than the corresponding plot for g_{sg} (see figure 4 in [12]) and the data (including those for $L = 2$) would appear to scale particularly well near T_{cg} . Once again, the value of ν_{cg} would appear to be very similar to that of $\nu_{sg} = 0.70 \pm 0.10$ [12].

We have appreciable uncertainty in the values of both T_{cg} and ν_{cg} . Furthermore, the increase in the reduced chiral-glass susceptibility for $T \leq 1.0$ is extremely sharp. As

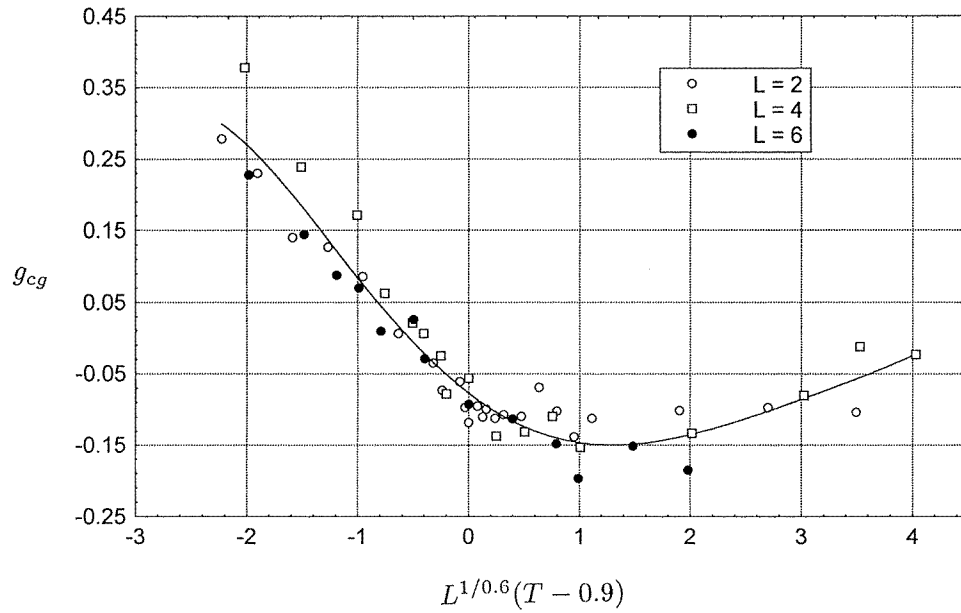


Figure 3. A scaling plot of the Binder parameter g_{cg} versus $L^{1/0.6}(T - 0.9)$. The curve is just a guide to the eye.

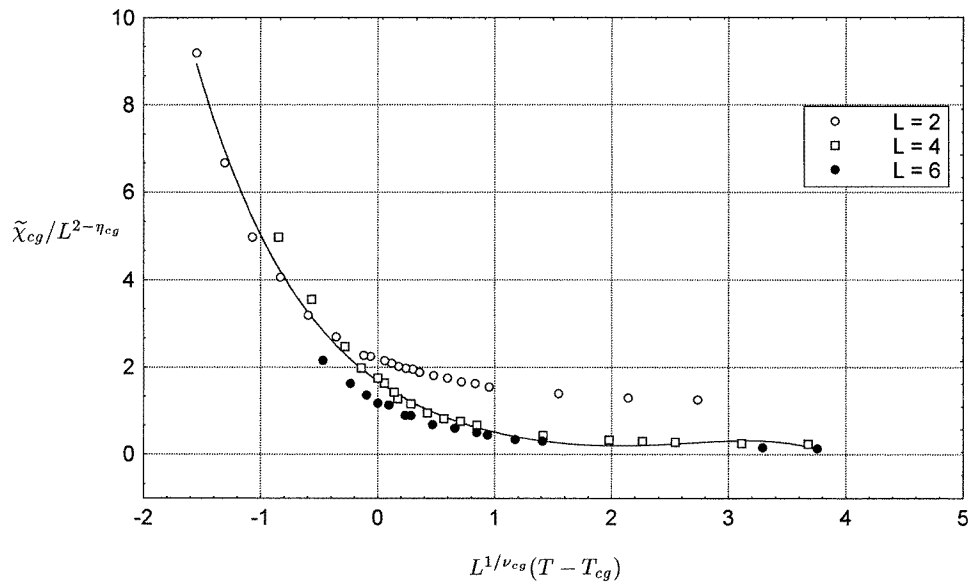


Figure 4. A scaling plot of $\tilde{\chi}_{cg}/L^{2-\eta_{cg}}$ against $L^{1/\nu_{cg}}(T - T_{cg})$ with $T_{cg} = 0.85$, $\nu_{cg} = 0.8$ and $\eta_{cg} = 0.25$ (see equation (11) and the text). The curve is just a guide to the eye.

a consequence, the critical region is very narrow. To obtain an estimate for the decay exponent, η_{cg} , we tried various different possible values of T_{cg} and ν_{cg} . The best such scaling plot is shown in figure 4 where we display $\tilde{\chi}_{cg}/L^{2-\eta_{cg}}$ against $L^{1/\nu_{cg}}(T - T_{cg})$ with $T_{cg} = 0.85$, $\nu_{cg} = 0.8$ and $\eta_{cg} = 0.25$.

Clearly, the $\tilde{\chi}_{cg}$ data for $L = 2$ do not scale all that well, especially for the higher temperatures. Nevertheless, we note that our estimate for η_{cg} is not incompatible with the value for η_{sg} found earlier [12].

5. Conclusion

To conclude, we have presented the results of a Monte Carlo simulation of the chiral-glass behaviour of the four-dimensional random-bond XY spin glass on small lattices ($L \leq 6$). By means of a finite-size scaling analysis of the data, we have estimated both the chiral transition temperature and the critical exponents. The chiral-glass values are very similar to their spin-glass counterparts for this model ($T_{cg} \approx T_{sg}$, $\nu_{cg} \approx \nu_{sg}$ and $\eta_{cg} \approx \eta_{sg}$). Hence, our results strongly suggest a simultaneous ordering of spin and chirality in four dimensions.

Further data on larger lattices ($L > 6$) is required to confirm both the transition temperature and the critical exponents.

Acknowledgment

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